OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4725

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$. **I** denotes the 2×2 identity matrix. Find

(i) $\mathbf{A} + 3\mathbf{B} - 4\mathbf{I}$, **[3]**

(ii) $\mathbf{AB}$. **[2]**

2 Prove by induction that, for $n \geqslant 1$, $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. **[5]**

3 By using the determinant of an appropriate matrix, find the values of k for which the simultaneous equations

$$kx + 8y = 1,$$
$$2x + ky = 3,$$

do not have a unique solution. **[3]**

4 Find $\displaystyle\sum_{r=1}^{2n}\left(3r^2 - \tfrac{1}{2}\right)$, expressing your answer in a fully factorised form. **[6]**

5 The complex number $1 + \mathrm{i}\sqrt{3}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by $|z - a| = |a|$ and $\arg(z - a) = \frac{1}{2}\pi$. **[6]**

6 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$, where $a \neq 1$. Find $\mathbf{C}^{-1}$. **[7]**

7 **(i)** Show that $\dfrac{1}{r-1} - \dfrac{1}{r+1} \equiv \dfrac{2}{r^2 - 1}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\displaystyle\sum_{r=2}^{n} \frac{2}{r^2 - 1}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=1000}^{\infty} \frac{2}{r^2 - 1}$. **[3]**

8 The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$.

(i) The diagram in the printed answer book shows the unit square $OABC$. The image of the unit square under the transformation represented by $\mathbf{X}$ is $OA'B'C'$. Draw and label $OA'B'C'$. **[3]**

(ii) The transformation represented by $\mathbf{X}$ is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B and state the matrices that represent them. **[4]**

9 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30\text{i}$.

(i) Write down the other root of the quadratic equation. **[1]**

(ii) Find the values of a and b. **[4]**

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. **[7]**

10 The cubic equation $x^3 + 3x^2 + 2 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \dfrac{1}{\sqrt{u}}$ to show that $4u^3 + 12u^2 + 9u - 1 = 0$. **[5]**

(ii) Hence find the values of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2}$ and $\dfrac{1}{\alpha^2\beta^2} + \dfrac{1}{\beta^2\gamma^2} + \dfrac{1}{\gamma^2\alpha^2}$. **[5]**

1 (i)	$\begin{pmatrix} 4 & 4a \\ 12 & 0 \end{pmatrix}$	B1		3**B** seen or implied
		B1		2 elements correct
		B1	3	Other 2 elements correct, a.e.f., including brackets
(ii)	$\begin{pmatrix} 4+4a & 3a \\ 4 & 1 \end{pmatrix}$	M1		Sensible attempt at matrix multiplication for **AB** or **BA**
		A1	2	Obtain correct answer
		5		
2		B1		Establish result true for $n = 1$ or 2
		M1*		Add next term to given sum formula
		DM1		Combine with correct denominator
		A1		Obtain correct expression convincingly
		A1	5	Specific statement of induction conclusion, provided 1st 4 marks earned
		5		
3	$k^2 - 16$	B1		Obtain correct det
		M1		Equate their det to 0
	$k = \pm 4$	A1	3	Obtain correct answers
		3		
4		M1		Express as sum of two series
	$3 \times \frac{1}{6} \times 2n(2n+1)(4n+1) - \frac{1}{2} \times 2n$	A1 A1		Each term correct a.e.f.
		M1		Attempt to factorise
	$2n^2(4n+3)$	A2	6	Completely correct answer, (A1 if one factor not found)
		6		
5 (i)	$\lvert a \rvert = 2$	B1		Correct modulus
	$\arg a = 60°, \frac{\pi}{3}, 1.05$	B1	2	Correct argument
(ii)		B1		Circle
		B1		Centre $(1, \sqrt{3})$
		B1		Through origin, centre $(\pm 1, \pm\sqrt{3})$ and another y intercept
		B1		Vertical line
		B1*		Through a or their centre, with +ve gradient
		DB1		Correct half line
			6	
		8		

6		M1		Show correct expansion process for 3×3 or multiplication of **C** and adj**C**
		M1		Correct evaluation of any 2×2
	det **C** = $\Delta = 5a - 5$	A1		Obtain correct answer
		M1		Show correct process for adjoint entries
	$\frac{1}{\Delta}\begin{pmatrix} 5 & -4 & 1 \\ -5 & 4a & -a \\ 5 & -3a-1 & 2a-1 \end{pmatrix}$	A1		Obtain at least 4 correct entries in adjoint
		A1		Obtain completely correct adjoint
		B1	**7**	Divide their adjoint by their determinant
		7		
7 (i)		B1	**1**	Obtain **given** answer correctly
(ii)		M1		Express at least 1st two and last two terms using (i)
		A1		1st two terms correct
		A1		Last two terms correct
		M1		Show that correct terms cancel
	$\frac{3}{2} - \frac{1}{n} - \frac{1}{(n+1)}$	A1	**5**	Obtain correct answer, a.e.f. in terms of n
(iii)		B1ft		Sum to infinity stated or implied or start at 1000 as in (ii)
		M1		S_∞ – their (ii) with n = 999 or 1000 or show correct cancelling
	$\frac{1999}{999000}$	A1	**3**	Obtain correct answer, a.e.f. (condone 0.002)
		9		
8 (i)		B1		(0, 3) seen
		B1		(3, 0) seen
		B1	**3**	Square with A ' B' and C' positioned correctly
(ii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ **or** $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1*		Reflection in $y = x$ or $y = -x$
		DB1		Correct matrix, dep on stating reflection
	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ **or** $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$	B1*		Enlargement scale factor 3 or s.f. -3
		DB1	**4**	Correct matrix, dep on stating enlargement **S.C. B2 for a pair of transformations consistent with their diagram.**
		7		

9	**(i)**	16 + 30i	B1	**1**	State correct value
	(ii)		M1		Use $a = -$ (sum of roots)
		$a = -32$	A1		Obtain correct answer
			M1		Use b = product of roots
		$b = 1156$	A1	**4**	Obtain correct answer
			M1		Substitute, expand and equate imag. parts
			A1		Obtain **a** = -32
			M1		Equate real parts
			A1		Obtain b = 1156
	(iii)		M1		Attempt to equate real and imaginary parts of $(p+iq)^2$ & 16 – 30i or root from (ii)
		$p^2 - q^2 = 16$ and $pq = -15$	A1		Obtain both results cao
			M1		Obtain quadratic in p^2 or q^2
			M1		Solve to obtain $p = (\pm)5$ or $q = (\pm)3$
			A1		Obtain 2 correct answers as complex nos
			M1		Attempt at all 4 roots
		$\pm\ (5 \pm 3i)$	A1	**7**	State other two roots as complex nos
			12		
10	**(i)**				
		$\frac{1}{u^{\frac{3}{2}}} + \frac{3}{u} + 2 = 0$	B1		Use substitution correctly
		EITHER	M1		Rearrange
			M1		Square
		$\frac{9}{u^2} + \frac{12}{u} + 4 = \frac{1}{u^3}$	A1		Obtain correct equation
		$4u^3 + 12u^2 + 9u - 1 = 0$	A1	**5**	Obtain **given** answer
		OR			
		e. g. $(2u^{3/2} + 3u^{1/2} + 1)(2u^{3/2} + 3u^{1/2} - 1) = 0$	M2		Multiply their equation in u by appropriate related expression
			A2		Obtain **given** answer
	(ii)		B1		Stated or imply that $u = \frac{1}{x^2}$
			M1		Use $-\frac{b}{a}$
		-3	A1		Obtain correct answer
			M1		Use $\frac{c}{a}$
		$\frac{9}{4}$	A1	**5**	Obtain correct answer
			10		